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NONSYMMETRIC BALLASTIC RANGE, HEIGHT, TIME-OF-FLIGHT  
AND OPTIMAL FLIGHT PATH ANGLE COMPUTATIONS  
WITH PROGRAMS FOR A HEWLETT-PACKARD 65 CALCULATOR

by

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March 1976

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  The purpose of this report is to provide the equations and HP-65 Programmable Calculator programs for computing ballistic range, height, time-of-flight, and particularly the flight path angle which maximizes ballistic ranges for non-symmetric launch and target positions. A no-atmosphere, non-rotating, spherical Earth is assumed.		



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NONSYMMETRIC BALLASTIC RANGE, HEIGHT, TIME-OF-FLIGHT  
AND OPTIMAL FLIGHT PATH ANGLE COMPUTATIONS  
WITH PROGRAMS FOR A HEWLETT-PACKARD 65 CALCULATOR

I. The Introduction and Apology.

The range of a ballistic missile over the surface of the Earth is a topic which is discussed in almost every text on astronautics (Reference 1, for example). Unfortunately, none of the available texts contained a needed procedure for the exact solution of the unsymmetric ballistic problem (defined in Section II); all procedures found were approximate and were based on a symmetric approximation [See References 2 and 3, for example]. The author of this report faced the dilemma of either (1) taking the time to solve the problem afresh, or (2) taking an inordinate longer time to do a thorough search to find a solution which might not be the one desired. The former course of action was taken and all due apologies are hereby extended to those that have published more elegant solutions.

All of the computational methods are outlined in Section III. Instructions and a sample problem for the Hewlett-Packard 65 Calculator (HP-65) are given in Section IV. This HP-65 programs are in Section V, and the development of the  $\beta_{opt}$  procedure is in the Appendix. Familiarity with Newtonian two-body theory is assumed.

## II. The Problem.

Basically, the problem is to compute the range,  $S$ , and time-of-flight,  $t_f$ , of a ballistic missile given the booster cutoff at height  $h_E$  after a vertical ascent, a cutoff velocity  $v_L$ , and a subsequent flight path angle  $\beta$  (Fig. 1). Given that a solution to the stated problem is available, the next natural question to ask is, "What flight path angle  $\beta_{opt}$  will obtain the maximum range  $S_{max}$ ?" The solution is fairly simple in the symmetric ballistic problem, i.e. when  $h_E = 0$ , and called symmetric because the launch point  $L$  and target point  $T$  are symmetric with the line of apses of the Keplerian orbit (i.e.  $\phi_1 = \phi_2$  in Fig. 1). The unsymmetric ballistic problem is that in which  $h_E \neq 0$ . Here we assume that  $h_E$  is given, the Earth with radius  $r_E$  is spherical, non-rotating, and has no atmosphere. The effect of the Earth's rotation may be calculated figuring how far the target has moved during the time of flight of the missile. Explicitly, we assume that Keplerian orbit is obtained at height  $h_E$  with terminal booster cutoff velocity  $v_L$  and flight path angle  $\beta$  (or  $\beta_L$ ), and the target  $T$  is at the Earth's surface.

The computation of range and time-of-flight for given  $h_E$ ,  $r_E$ ,  $v_L$ , and  $\beta$  can easily be obtained from the equations of elliptical orbits. The determination of  $\beta_{opt}$  is somewhat complicated, however, and requires an iterative procedure. In all of the developments we assume that  $L$  and  $T$  are separated by the line of apses (which implies that  $0 < \beta \leq \frac{\pi}{2}$ ).



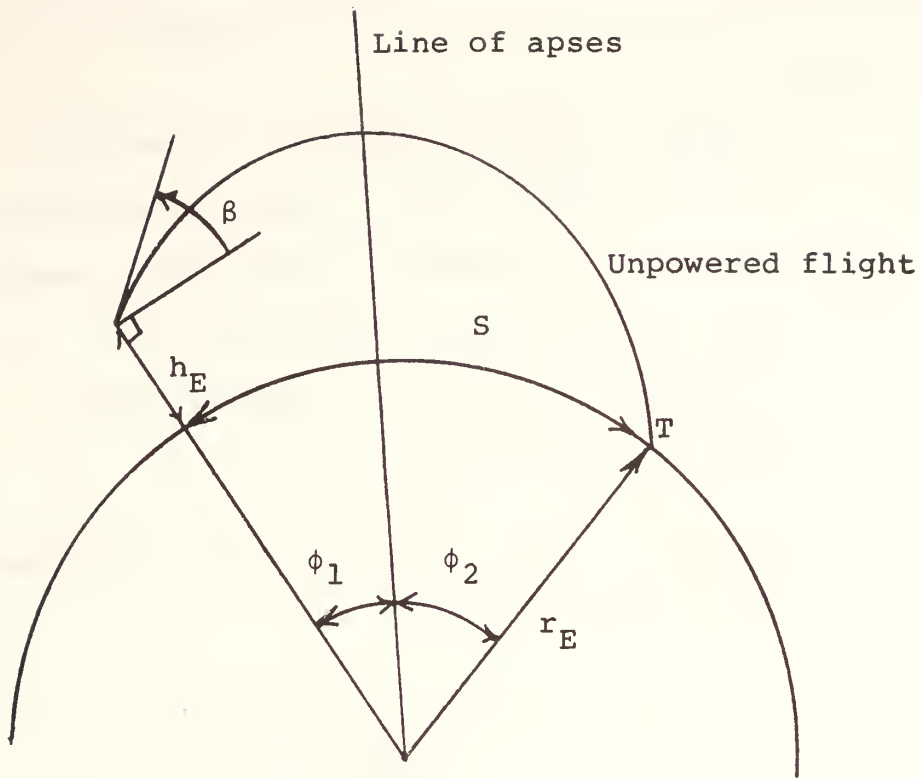


FIGURE 1

### III. The Computational Procedures.

#### A. Range

1. Let  $r_L$  and  $r_T$  denote the distance of the launch point L and target point T from the center of the Earth. Usually we let  $r_L = r_E + h_E$  and  $r_T = r_E$  although  $r_L$  and  $r_T$  need not be so defined in general.

2. From  $r_L$  and  $v_L$ , compute the semimajor axis of the ellipse,

$$a = \frac{\mu}{\left(\frac{2\mu}{r_L}\right) - v_L^2},$$

where  $\mu = k_E^2$  and  $k_E$  is the (Gaussian) gravitational constant for the Earth. We have assumed that the mass of the missile is negligible with respect to the mass of the Earth.

3. Using the flight path angle  $\beta$ , compute the elliptical semiparameter

$$p = (r_L v_L \cos \beta)^2 / \mu.$$

4. Compute the eccentricity

$$e = \sqrt{1 - \frac{p}{a}}.$$

5. Compute the true anomaly of the launch point

$$f_L = \cos^{-1} \left[ \frac{1}{e} \left( \frac{p}{r_L} - 1 \right) \right]$$

where  $0 \leq f_L \leq \pi$ . Note that the line of apses can be said to have true anomaly  $f = \pi$ .

6. Compute the true anomaly of the target point

$$f_T = 2\pi - \cos^{-1} \left[ \frac{1}{e} \left( \frac{p}{r_T} - 1 \right) \right]$$

7. The surface range of the missile is then

$$S = r_E (f_T - f_L) .$$

## B. Time-of-Flight

1. Compute the eccentric anomaly of the launch point,

$$E_L = \cos^{-1} \left[ \frac{1}{e} \left( 1 - \frac{r_L}{a} \right) \right]$$

where  $0 \leq E_L \leq \pi$ .

2. Compute the eccentric anomaly of the target point,

$$E_T = 2\pi - \cos^{-1} \left[ \frac{1}{e} \left( 1 - \frac{r_T}{a} \right) \right]$$

where  $\pi \leq E_T \leq 2\pi$ .

3. Using Keplers' equation, compute the time-of-flight

$$t_f = \frac{a^{3/2}}{\sqrt{\mu}} \left[ (E_T - E_L) - e(\sin E_T - \sin E_L) \right] .$$

C. Height of Trajectory & Circular Velocity

1. The maximum height of the trajectory above the Earth's surface for any  $h_E$ ,  $v_L$ , and  $\beta$  is given by

$$h_{\max} = h = a(1 + e) - r_E$$

2. Circular Velocity

The velocity required for a circular orbit at  $r_L$  is

$$v_c = \sqrt{\mu/r_L} .$$

$v_c$  will be used in Section III D.

D. Optimum Flight Path Angle & Maximum Range

For a specified  $h_E$  and  $v_L$ , a value of  $\beta$  can be found such that

$$S_{\max} = S(\beta_{\text{opt}}) = \max_{\beta} S(\beta) .$$

In this unconstrained optimization problem

we set  $\frac{\partial S}{\partial \beta} \Big|_{\beta = \beta_{\text{opt}}} = 0$  and solve

for  $\beta_{\text{opt}}$ . The relationship between  $S$  and  $\beta$  is given by the computational formulas for the range in part A. Although an explicit formula for  $\beta$  is unobtainable, the following exact relationship is useful:

$$(1) \quad \cos^2 \beta_{\text{opt}} = \frac{1}{(2-\alpha^2)} \left\{ 1 + \frac{\left[ 1 - \left( \frac{r_L}{r_T} \right) (2-\alpha^2) \cos^2 \beta_{\text{opt}} - \frac{2\mu}{v_L^2} \left( \frac{1}{r_L} - \frac{1}{r_T} \right) \right] \sin \beta_{\text{opt}}}{\sqrt{1 - \left( \frac{r_L}{r_T} \right)^2 \cos^2 \beta_{\text{opt}} - \frac{2\mu}{v_L} \left( \frac{1}{r_L} - \frac{1}{r_T} \right)}} \right\}$$

where  $\alpha = \frac{v_L}{v_c}$ .

In the symmetric ballistic problem  $r_L = r_T$  ;  
the equation for  $\beta_{\text{opt}}$  then reduces to

$$(2) \quad \cos^2 \beta_{\text{opt}} = \frac{1}{2-\alpha^2},$$

which is the result given in References 1, 2, and 3.

In the unsymmetric case Equation (1) is used to find  $\beta_{\text{opt}}$  as follows:

1. Solve (2) for  $\beta_{\text{opt}}$ .
2. Substitute  $\beta_{\text{opt}}$  into the right hand side of (1) to obtain a new value for  $\beta_{\text{opt}}$ .

3. Average the last two values of  $\beta_{opt}$  to obtain a new value of  $\beta_{opt}$  and return this value of  $\beta_{opt}$  to step 2.
4. It has been found that five iterations are sufficient to obtain a reliable value for  $\beta_{opt}$ .
5. Using  $\beta_{opt}$ , compute the maximum range  $S_{max} = S(\beta_{opt})$  using procedure A.

#### IV. The Illustrative Example.

The HP-65 program consists of four magnetic cards which are used as follows:

Card 1: This card must be entered first. It is used to set the two constants  $r_E$  and  $\mu$  for any choice of four distance units and any choice of four time units. The distance units are kilometers, meters, statute miles, and nautical miles; the distance unit is entered by pressing key A, B, C, or D, respectively. The display shows the equatorial Earth radius  $r_E$  in the respective unit. The time units are seconds, minutes, hours, and days; the time unit is entered by pressing key A, B, C, or D, respectively. The display shows the constant  $\mu$  in the appropriate (distance)<sup>3</sup>/(time)<sup>2</sup> unit. Note - after entering program Card 1, the distance unit must be entered before the time unit is entered since the keys A through D perform a dual function. The basic definitions used are:

$$1 \text{ statute mile} = 1.609344 \text{ kilometers(km)}$$

$$1 \text{ nautical mile} = 1.852 \text{ km}$$

$$r_E = 6378.160 \text{ km}$$

$$\mu = 398603 \text{ km}^3/\text{sec}^2$$

Card 2: This card computes  $S$ ,  $h_{\max}$ , and  $v_C$  given  $h_E$ ,  $v_L$  and  $\beta$ . It must be used after the

execution of Card 1 and it may be used before and/or after the execution of Cards 3 and 4. If it is used after Card 4 to compute  $S_{\max}$ , then  $\beta_{\text{opt}}$  as computed by Card 4 need not be re-entered. Otherwise, in any order: enter  $h_E$  and press A, enter  $v_L$  and press B, enter  $\beta$  and press C. To find the range S, press E. Then to find  $h_{\max}$ , press R/S; and then to find  $v_C$ , press R/S again. Note: If  $h_E$ ,  $v_L$ , and  $\beta$  have been entered using Card 3, then they do not have to be reentered when using Card 2. Also, any new value of  $h_E$ ,  $v_L$ , or  $\beta$  can be entered and the range computed without having to re-enter unchanged values.

Card 3: This card computes the time of flight,  $t_f$ . It must be used after the execution of Card 1 and it may be used before and/or after the execution of Cards 2 and 4. If it is used after Card 4, then  $\beta_{\text{opt}}$  as computed by Card 4 need not be re-entered; if it is used after Card 2, then only new quantities need be entered. Otherwise, in any order: enter  $h_E$  and press A, enter  $v_L$  and press B, enter  $\beta$  and press C. To find the time of flight, press E.



Card 4: This card computes  $\beta_{opt}$ . It must be used after  $h_E$  and  $v_L$  have been entered from either Card 2 or Card 3. No entries are required. To compute  $\beta_{opt}$ , press any key A through E; five iterations will be performed and the resulting value of  $\beta_{opt}$  will be displayed. To perform five more iterations, press R/S. To monitor the intermediate iterations press keys f SF2. Each intermediate result will be displayed. To continue, press R/S. To disable the monitoring, press keys f<sup>-1</sup> SF2.

Using the four program cards as described above find the range and time-of-flight for a ballistic missile that has a velocity of 10000 knots at a cut-off altitude of 50 nautical miles for a flight path angle of  $30^\circ$  and  $40^\circ$ . Also, find the maximum possible range and time-of-flight for the same missile.

Proceed as follows:

<u>Step</u>	<u>Instruction</u>	<u>Data</u>	<u>Key</u>	<u>Display</u>
1	Enter Card 1			
2	Distance unit is n.mi.		D	$r_E = 3443.93 \text{ n.mi.}$
3	Time unit is hours		C	$\mu = 8.13247 \times 10^{11}$
4	Enter Card 2			
5	Enter cutoff altitude	50	A	$r_L = 3493.93 \text{ n.mi.}$
6	Enter cutoff velocity	10000	B	
7	Enter flight path angle	30	C	
8	Compute range		E	$S = 1928.08 \text{ n.mi.}$

<u>Step</u>	<u>Instruction</u>	<u>Data</u>	<u>Key</u>	<u>Display</u>
8a	Optional: Display $h_{\max}$		R/S	$h_{\max} = 344.74 \text{ n.mi.}$
8b	Optional: Display $v_c$		R/S	$v_c = 15256.47 \text{ knot}$
9	Enter flight path angle	40	C	
10	Compute range		E	$S = 1956.60 \text{ n.mi.}$
11	Enter Card 3			
12	Compute time-of-flight Note: $h_E = 50$ , $v_L = 10000$ , and $\beta = 40^\circ$ since no further entries have been made.		E	$t_f = 0.3043 \text{ hrs.}$
13	Enter flight path angle	30	C	
14	Compute time-of-flight		E	$0.2504 \text{ hrs.}$
15	Enter Card 4			
16	Compute $\beta_{\text{opt}}$ (5 iterations)		E	$\beta_{\text{opt}} = 36.0895$
17	Make 5 more iterations to be sure		R/S	$\beta_{\text{opt}} = 36.0895$
18	Enter Card 2			
19	Compute $S_{\max}$		E	$S_{\max} = 1973.95 \text{ n.m.}$
20	Enter Card 3			
21	Compute time-of-flight for maximum range.		E	$t_f(\beta_{\text{opt}}) = 0.2846$

V. The Program Listings.

CARD 1

Set units of distance and time display  $r_E$  and  $\mu$

Key Entry	Code Shown	Key Entry	Code Shown
LBL	23	3	03
B	12	7	07
EEX	43	8	08
3	03	.	83
CHS	42	1	01
GTO	22	6	06
0	00	X	71
LBL	23	STO 4	3304
C	13	R/S	84
10 1	01	60 LBL	23
.	83	B	12
6	06	6	06
0	00	0	00
9	09	GTO	22
3	03	1	01
4	04	LBL	23
4	04	C	13
GTO	22	3	03
0	00	6	06
20 LBL	23	70 0	00
D	14	0	00
1	01	GTO	22
.	83	1	01
8	08	LBL	23
5	05	D	14
2	02	8	08
GTO	22	6	06
0	00	4	04
LBL	23	0	00
30 A	11	80 0	00
1	01	GTO	22
LBL	23	1	01
0	00	LBL	23
g	35	A	11
1/x	04	1	01
ENTER	41	LBL	23
ENTER	41	1	01
ENTER	41	ENTER	41
X	71	X	71
40 X	71	90 RCL 1	3401
3	03	X	71
9	09	STO 1	3301
8	08	R/S	84
6	06	g NOP	3501
0	00	g NOP	3501
3	03	"	3501
X	71	"	3501
STO 1	3301	"	3501
CL x	44	"	3501
50 6	06	100 "	3501

CARD 2

Compute range,  $h_{max}$ ,  $v_c$

Key Entry	Code Shown	Key Entry	Code Shown
RCL 4	3404	X	71
+	61	1	01
STO 3	3303	-	51
R/S	84	CHS	42
LBL	23	f	31
B	12	$\sqrt{\quad}$	09
STO 2	3302	STO 6	3306
R/S	84	RCL 8	3408
LBL	23	RCL 3	3403
10 C	13	60 D	14
STO 7	3307	RCL 8	3408
R/S	84	RCL 4	3404
LBL	23	D	14
D	14	+	61
g	35	g	35
RAD	42	$\pi$	02
$\div$	81	ENTER	41
1	01	+	61
-	51	g x $\leftrightarrow$ y	3507
20 RCL 6	3406	70 -	51
$\div$	81	RCL 4	3404
f-1	32	x	71
COS	05	R/S	84
RTN	24	RCL 6	3406
LBL	23	1	01
E	15	+	61
g	35	RCL 5	3405
DEG	41	$\div$	81
30 RCL 7	3407	80 RCL 4	3404
f	31	-	51
COS	05	R/S	84
RCL 2	3402	RCL 1	3401
X	71	RCL 3	3403
RCL 3	3403	$\div$	81
X	71	f	31
ENTER	41	$\sqrt{\quad}$	09
X	71	R/S	84
RCL 1	3401	GTO	22
$\div$	81	E	15
40 STO 8	3308	90 g NOP	3501
2	02	g NOP	3501
RCL 3	3403	"	3501
$\div$	81	"	3501
RCL 2	3402	"	3501
ENTER	41	"	3501
X	71	"	3501
RCL 1	3401	"	3501
$\div$	81	"	3501
-	51	"	3501
50 STO 5	3305	100 "	3501

NOTE: Before putting a program into memory, after switching to W/PGM mode, press f and then PRGM to clear memory.

CARD 3

Compute time-of-flight

Key	Code		Key	Code
Entry	Shown		Entry	Shown
RCL 4	3404		-	51
+	61		STO 5	3305
STO 3	3303		X	71
R/S	84		1	01
LBL	23		-	51
B	12		CHS	42
STO 2	3302		f	31
R/S	84		√	09
LBL	23		STO 6	3306
C	13	60	g	35
STO 7	3307		π	02
R/S	84		ENTER	41
LBL	23		+	61
D	14		RCL 4	3404
g	35		D	14
RAD	42		-	51
RCL 5	3405		ENTER	41
X	71		f	31
1	01		SIN	04
-	51	70	RCL 6	3406
CHS	42		X	71
RCL 6	3406		-	51
÷	81		RCL 3	3403
f <sup>-1</sup>	32		D	14
COS	05		ENTER	41
RTN	24		f	31
LBL	23		SIN	04
E	15		RCL 6	3406
g	35		X	71
DEG	41	80	-	51
RCL 7	3407		-	51
f	31		RCL 5	3405
COS	05		÷	81
RCL 2	3402		RCL 1	3401
X	71		RCL 5	3405
RCL 3	3403		X	71
X	71		f	31
ENTER	41		√	09
X	71		÷	81
RCL 1	3401	90	R/S	84
÷	81		GTO	22
STO 8	3308		E	15
2	02		g NOP	3501
RCL 3	3403		g NOP	3501
÷	81		g NOP	3501
RCL 2	3402		"	3501
ENTER	41		"	3501
X	71		"	3501
RCL 1	3401		"	3501
÷	81	100	"	3501

CARD 4

Compute maximizing flight path angle  $\beta_{opt}$

Key	Code		Key	Code
Entry	Shown		Entry	Shown
2	02		SIN	04
RCL 2	3402		X	71
ENTER	41		RCL 5	3405
X	71		RCL 3	3403
STO 5	3305		RCL 4	3404
RCL 3	3403		÷	81
X	71		RCL 7	3407
RCL 1	3401		f	31
÷	81		COS	05
-	51	60	X	71
STO 6	3306		ENTER	41
RCL 4	3404		X	71
g	35		-	51
1/x	04		f	31
RCL 3	3403		√	09
g	35		÷	81
1/x	04		1	01
-	51		+	61
RCL 5	3405		RCL 6	3406
÷	81	70	÷	81
RCL 1	3401		f	31
X	71		√	09
ENTER	41		f <sup>-1</sup>	32
+	61		COS	05
1	01		f	31
+	61		TF 2	81
STO 5	3305		R/S	84
g	35		g NOP	3501
DEG	41		RCL 7	3407
0	00	80	gx↔y	3507
STO 7	3307		STO 7	3307
5	05		0	00
STO 8	3308		g x=y	3523
LBL	23		GTO	22
0	00		0	00
RCL 5	3405		+	61
RCL 3	3403		+	61
RCL 4	3404		2	02
÷	81		÷	81
RCL 6	3406	90	STO 7	3307
X	71		g	35
RCL 7	3407		DSZ	83
f	31		GTO	22
COS	05		0	00
X	71		R/S	84
gLSTx	3500		5	05
X	71		STO 8	3308
-	51		GTO	22
RCL 7	3407		0	00
f	31	100	g NOP	3501

NOTE: Before putting a program into memory, after switching to W/PGM mode, press f and then PRGM to clear memory.

VI. The References.

1. Berman, Arthur I., The Physical Principles of Astronautics, John Wiley and Sons, Inc. 1961.
2. Seifert, Howard (editor), Space Technology, John Wiley and Sons, Inc. 1959.
3. Wyckoff, Robert C., Private Communication, 1975.

VII. The Appendix.

As indicated in section IIID, the maximum range for a specified cutoff altitude  $h_E$  and cutoff velocity  $v_L$  is found by the following unconstrained optimization:

$$S_{\max} = S(\beta_{\text{opt}}) = \max S(\beta)$$

$$0 < \beta \leq \frac{\pi}{2}$$

Using the equations in section IIIA, we want to find  $\beta_{\text{opt}}$  such that

$$\left. \frac{\partial S}{\partial \beta} \right|_{\beta = \beta_{\text{opt}}} = r_E \left. \frac{\partial}{\partial \beta} (f_T - f_L) \right|_{\beta = \beta_{\text{opt}}} = 0 .$$

We find that

$$\frac{\partial}{\partial \beta} (f_T - f_L) = (\gamma_T - \gamma_L) \frac{\partial p}{\partial \beta} ,$$

where

$$\gamma_i = \frac{1}{\sqrt{1 - \frac{1}{e^2} \left(\frac{p}{r_i}\right)^2 - 1}} \left[ \frac{1}{er_i} + \frac{1}{2ae^3} \left(\frac{p}{r_i} - 1\right) \right], \quad i=T, L .$$

For the derivative of  $f_T - f_L$  with respect to  $\beta$  to vanish at  $\beta = \beta_{\text{opt}}$ , we must have either:

$$1. \quad (\gamma_T - \gamma_L)_{\beta = \beta_{\text{opt}}} = 0$$

or

$$2. \quad \left. \frac{\partial p}{\partial \beta} \right|_{\beta = \beta_{\text{opt}}} = 0$$

We find that

$$\frac{\partial p}{\partial \beta} = - \frac{r_L^2 v_L^2 \sin 2\beta}{\mu} = 0$$

for  $\beta = 0, \pm \pi/2, \pm \pi, \text{ etc.}$  Since we require that the launch point and the target point be separated by the line of apsides, we require that  $\theta < \beta \leq \frac{\pi}{2}$ . This requirement is imposed because we must properly interface the discontinuity caused by the principal angle in the arc cosine used in steps 5 and 6 of section III A. Thus  $\beta = \pi/2$  is the only angle of interest for which  $\partial p / \partial \beta = 0$ , and this angle clearly gives rise to a minimum in range since the trajectory of the missile is vertical (straight up, and then straight down).

The value of  $\beta$  which maximizes  $S$  is found from the requirement that

$$(\gamma_T - \gamma_L)_{\beta = \beta_{\text{opt}}} = 0 .$$

This equation has defied attempts to find an analytic solution, but it can be rearranged and displayed as equation 1 in section III D, where the solution procedure is detailed.



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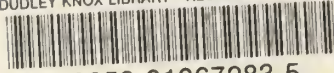
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